

New Universality Classes for Quantum Critical Behavior

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Abstract

We use the epsilon expansion to explore a new universality class of second order quantum phase transitions associated with a four-dimensional Yukawa field theory coupled to a traceless Hermitean matrix scalar field. We argue that this class includes four-fermi models in $2 < D < 4$ dimensions with $SU(N_F) \times U(N)$ symmetry and a $U(N)$ scalar, $SU(N_F)$ iso-vector 4-fermi coupling. The epsilon expansion indicates that there is a second order phase transition for $N \geq N^*(N_F)$, where $N^*(N_F) \simeq .27N_F$ if $N_F \rightarrow \infty$.

There are several scenarios in field theory where quantum critical phenomena play a role. They are intimately associated with renormalization theory and are important in modifications of the standard model using technicolor [1] or models with composite Higgs fields [2]-[6]. Quantum critical phenomena are associated with phase transitions which are achieved, typically at zero temperature, by tuning mechanical parameters such as particle masses or coupling constants to critical values. Classical critical phenomena, on the other hand, occur at non-zero temperature and are associated with phase transitions where temperature or some other thermodynamic variable is the quantity which must be tuned.

An important concept in the study of second order phase transitions is universality. A phase transition is characterized by the number of degrees of freedom which become massless at the critical point, as well as the symmetry of the system and the symmetry breaking pattern associated with the transition. In classical critical phenomena, at finite temperature, fermions decouple from infrared behavior because of their half-odd-integer Matsubara frequency. Thus, all universality classes are represented by Bose field theories. In quantum critical behavior, fermions can be massless at a phase transition and therefore they must be included in the effective low energy field theories which represent universality classes.

A useful tool for the study of phase transitions is the Wilson-Fisher epsilon expansion where the beta function is computed in a $4 - \epsilon$ dimensional effective field theory and second order phase transitions are characterized by the infrared stable fixed points of the renormalization group flow. If the beta function exhibits such fixed points and if the coupling constants have initial conditions within their domain of attraction, there is a second order transition. If no infrared stable fixed points exist then if there is a phase transition it must be of first order, driven by fluctuations [7]-[9].

The epsilon expansion has been used to show that the $O(N)$ vector model in $4 - \epsilon$ dimensions has a second order phase transition with computable critical exponents [10]. It represents a universality class which contains the $O(N)$ nonlinear sigma model in dimensions between 2 and 4. The sigma model is renormalizable only in the $1/N$ expansion [11] and represents the continuum limit of generalized magnetic systems. It has also been argued [12]-[14] that the four dimensional Yukawa- ϕ^4 theory in $4 - \epsilon$ dimensions represents the universality class of the Gross-Neveu model in dimensions between 2 and 4. The latter is also renormalizable only in the $1/N$ expansion [15, 16] and the critical exponents have

been computed to high order [17]-[19]. The $O(N)$ model has one relevant coupling constant in the 4-dimensional theory, so the fixed point of order ϵ is automatically infrared stable and the phase transition is of second order. When there is more than one relevant interaction in the $4 - \epsilon$ dimensional theory, and therefore a multi-dimensional coupling constant flow, the generic situation is that fixed points of the flow are not infrared stable and the phase transitions are first order. The classic example of this is the massless scalar electrodynamics studied by Coleman and Weinberg [8]. There, in the two-dimensional plane of electromagnetic and Higgs self-coupling there are no infrared stable fixed points. Another example occurs in the complex matrix scalar field theory with $SU(N_L) \times SU(N_R)$ symmetry which represents the universality class of the finite temperature chiral transition in QCD [20]. There are two coupling constants for the renormalizable interactions $\text{tr}(M^\dagger M)^2$ and $(\text{tr}M^\dagger M)^2$ and when the matrices are larger than 2×2 the phase transition is first order. When the matrices are 2×2 the model is equivalent to a vector theory with one coupling constant and the phase transition is second order. Similar reasoning has been used to argue that a $4 - \epsilon$ -dimensional traceless Hermitean matrix scalar field theory with $SU(N_F)$ symmetry, which also has two relevant couplings $\text{tr}\phi^4$ and $(\text{tr}\phi^2)^2$, has a second order phase transition only when $N_F = 2$ and has a fluctuation induced first order phase transition when $N_F > 2$ [21].

In this Letter, we shall show that when a Yukawa coupling to fermions is introduced to the Hermitean matrix theory, the existence of a nontrivial Yukawa fixed point tends to make the fixed points of the matrix theory infrared stable. For a given N_F , we always find a second order phase transition if the number of fermions in the fundamental representation is large enough, asymptotically $N > .27N_F$ for large N, N_F , and first order otherwise. We shall also argue that the Yukawa-matrix model represents the universality class of a second order quantum phase transition in a matrix-coupled four-fermi theory in $2 < D < 4$ dimensions. We use this equivalence to argue that, for large enough N , the epsilon expansion is valid for epsilon of order 1.

The Euclidean action for the $4 - \epsilon$ -dimensional Yukawa model is

$$\begin{aligned} S = & \int d^{4-\epsilon}x \left(\bar{\psi}_\alpha^a \left(\delta_{\alpha\beta} \vec{\gamma} \cdot \vec{\nabla} + \frac{\pi\mu^{\epsilon/2}y}{\sqrt{N_F N}} \phi_{\alpha\beta} \right) \psi_\beta^a \right. \\ & \left. + \frac{1}{2} \text{tr}_F \vec{\nabla} \phi \cdot \vec{\nabla} \phi + \frac{8\pi^2\mu^\epsilon}{4!} \left(\frac{g_1}{N_F^2} (\text{tr}_F \phi^2)^2 + \frac{g_2}{N_F} \text{tr}_F \phi^4 \right) \right), \end{aligned} \quad (1)$$

where $a, b = 1, \dots, N$, $\alpha, \beta = 1, \dots, N_F$, $\phi_{\alpha\beta}$ is a traceless Hermitean $N_F \times N_F$ scalar matrix field. ψ_α^a is a complex Dirac spinor. Here we have included all vertices which are compatible with the unitary symmetry, $\phi \rightarrow U\phi U^\dagger$, $\psi \rightarrow U\psi$, $\bar{\psi} \rightarrow \bar{\psi}U^\dagger$ with $U \in SU(N_F)$ and which are renormalizable in 4 dimensions. We have scaled the coupling constants so that in the large N_F limit planar graphs dominate and fermion loops are suppressed and in the large N limit the bubble graphs dominate.

The beta functions of this model for the case $y = 0$ was computed to one-loop order in [21]. The beta function for the model with $y \neq 0$ is readily obtained to the same order:

$$\beta_1 = -\epsilon g_1 + \frac{N_F^2 + 7}{6N_F^2} g_1^2 + \frac{2N_F^2 - 3}{3N_F^2} g_1 g_2 + \frac{N_F^2 + 3}{2N_F^2} g_2^2 + \frac{1}{2N_F} y^2 g_1, \quad (2)$$

$$\beta_2 = -\epsilon g_2 + \frac{2}{N_F^2} g_1 g_2 + \frac{N_F^2 - 9}{3N_F^2} g_2^2 - \frac{3}{8NN_F} y^4 + \frac{1}{2N_F} y^2 g_2, \quad (3)$$

$$\beta_y = -\frac{\epsilon}{2} y + \frac{2NN_F + N_F^2 - 3}{16N_F^2 N} y^3. \quad (4)$$

Also at one-loop order, the scaling dimension of the scalar field is

$$\Delta_B = \frac{2 - \epsilon}{2} + \frac{1}{8N_F} y^2 + \dots, \quad (5)$$

and that of the fermion field is

$$\Delta_F = \frac{3 - \epsilon}{2} + \frac{1}{32} \frac{N_F^2 - 1}{N_F^2 N} y^2 + \dots \quad (6)$$

There will be a second order phase transition when the beta function has infrared stable fixed points. This occurs where $\beta_i(g) = 0$ and where all eigenvalues of the matrix $\partial\beta_i/\partial g_j$ are positive. The trivial fixed point $y_0^* = 0$ of the Yukawa coupling always coincides with a negative eigenvalue of the stability matrix. It could therefore be ultraviolet stable, but not infrared stable. Thus, to find infrared fixed points, we take the nonzero fixed point of the Yukawa coupling,

$$y_*^2 = \frac{8N_F^2 N}{2NN_F + N_F^2 - 3} \epsilon. \quad (7)$$

This coupling is small when ϵ is small.

In the large N limit, the beta functions have an infrared stable fixed point at Yukawa coupling (7) and scalar self-couplings

$$g_1^* = -\frac{18(N_F^2 + 3)}{N^2} \epsilon + \dots, \quad g_2^* = \frac{6N_F}{N} \epsilon + \dots \quad (8)$$

In this regime, the scalar self couplings are small and the one-loop perturbation theory is accurate. The scaling dimensions of the spinor and scalar fields can be computed to leading order in ϵ and $1/N$ at the fixed points

$$\Delta_F = \frac{3 - \epsilon}{2} + \frac{N_F^2 - 1}{8NN_F}\epsilon + \dots, \quad (9)$$

$$\Delta_B = 1 + \dots \quad (10)$$

These critical exponents depend on both N and N_F and therefore each value of these parameters specifies a different universality class.

We have solved for the fixed points numerically for general values of N and N_F . The asymptotic form of the fixed points in (7) and (8) continued to lower N are depicted for a few values of N_F in Fig. 1. A critical value of N , $N^*(N_F)$, can be defined for every N_F , such that an infrared stable fixed point exists at $N \geq N^*(N_F)$. If $N_F \rightarrow \infty$ then $N^*(N_F) \simeq 0.27N_F$. This asymptotic limit is reached through values of $N^*(2) = 34$, $N^*(4) = 8$, $N^*(8) = 4$, $N^*(80) = 22$, $N^*(800) = 216$.

An example of a dynamical system whose universality class is described by (1) is the four-fermi theory [22]

$$S = \int d^Dx \left(\bar{\psi}_\alpha^\alpha \vec{\gamma} \cdot \vec{\nabla} \psi_\alpha^\alpha - \frac{\lambda}{N} \bar{\psi}_\alpha^\alpha T_{\alpha\beta}^A \psi_\beta^\alpha \bar{\psi}_\gamma^\beta T_{\gamma\delta}^A \psi_\delta^\beta \right), \quad (11)$$

where $2 < D < 4$, $\alpha, \beta, \dots = 1, \dots, N_F$, $a, b, \dots = 1, \dots, N$ and ψ is a 4-component Dirac spinor. T^A are generators of $SU(N_F)$ in the fundamental representation, normalized so that $\text{tr}T^A T^B = \delta^{AB}/2$. This model has the same symmetries as (1): C , P , T , as well as discrete chiral symmetry and global $SU(N_F) \times U(N)$. [23] The parity and discrete chiral transformation of the four component fermions can be defined in such a way that the mass operator $\bar{\psi}\psi$ transforms as a scalar under parity and changes sign under a discrete chiral transformation [24]. The phase transition that we shall study generates fermion mass through spontaneous breaking of the $SU(N_F)$ and discrete chiral symmetry. The order parameter is $\hat{\phi}_{\alpha\beta} \equiv (-\lambda/N) < \bar{\psi}_\alpha^\alpha \psi_\beta^\alpha - \frac{1}{N_F} \delta_{\alpha\beta} \bar{\psi}_\gamma^\alpha \psi_\gamma^\alpha >$.

Four-fermi models with isoscalar couplings are known to be renormalizable in the large N expansion [15, 16, 25, 26] when the coupling is tuned to a critical value where a second order phase transition takes place. Existence of a phase transition has also been demonstrated in the conformal bootstrap approach in isoscalar models [27]. Their arguments can be extended

to the model (11) in a straightforward way. We introduce an $N_F \times N_F$ auxiliary field, ϕ ,

$$S = \int d^D x \left(\bar{\psi}_\alpha^a (\delta_{\alpha\beta} \vec{\gamma} \cdot \vec{\nabla} + \phi_{\alpha\beta}) \psi_\beta^a + \frac{N}{2\lambda} \text{tr}_F \phi^2 \right), \quad (12)$$

where $\text{tr}_F \phi = 0$. Here, the fermions can be integrated to obtain the scalar field theory

$$S_{eff} = -N \text{TR} \ln \left(\vec{\gamma} \cdot \vec{\nabla} + \phi \right) + \int d^D x \frac{N}{2\lambda} \text{tr}_F \phi^2, \quad (13)$$

which can be studied perturbatively in the $1/N$ expansion. (Here TR means trace in function space and over indices.) The effective scalar propagator for the large N expansion,

$$\begin{aligned} \Delta_{\alpha\beta\gamma\delta}(p) &\equiv \langle \phi_{\alpha\beta}(p) \phi_{\gamma\delta}(-p) \rangle = \left(\delta_{\alpha\delta} \delta_{\beta\gamma} - \frac{1}{N_F} \delta_{\alpha\beta} \delta_{\gamma\delta} \right) \\ &\times \frac{1}{N} \left(\frac{1}{\lambda} - \frac{1}{\lambda_c} - \frac{4\Gamma(1-D/2)[\Gamma(D/2)]^2 p^{D-2}}{(4\pi)^{D/2} \Gamma(D-1)} \right)^{-1}. \end{aligned} \quad (14)$$

λ_c is the critical coupling which is of order Λ^{2-D} where Λ is the large momentum cutoff. The propagator is scale invariant when λ is tuned to λ_c . This is the value of the coupling at which the phase transition takes place. When λ has this critical value, the scaling dimensions of the scalar and spinor fields are defined by the behavior of the fermion and scalar propagators,

$$S_{\alpha\beta}(p) \sim \delta_{\alpha\beta} \frac{1}{-i\vec{\gamma} \cdot \vec{p}} |p|^{2\Delta_F+1-D}, \quad (15)$$

$$\Delta_{\alpha\beta\gamma\delta}(p) \sim \left(\delta_{\alpha\delta} \delta_{\beta\gamma} - \frac{1}{N_F} \delta_{\alpha\beta} \delta_{\gamma\delta} \right) |p|^{2\Delta_B-D}, \quad (16)$$

respectively. From this and computation of the spinor self-energy to leading order in $1/N$ and (14), we deduce

$$\Delta_F = \frac{D-1}{2} - \frac{\Gamma(D-1)(N_F^2-1)}{8N_F N \Gamma(1-D/2)[\Gamma(D/2)]^2 \Gamma(D/2-1)} + \dots, \quad (17)$$

$$\Delta_B = 1 + \dots. \quad (18)$$

If we set $D = 4-\epsilon$ in (17) and expand to first order in ϵ , the scaling dimension of the fermion in the four-fermi model is identical with that in the Yukawa-matrix model at the infrared stable fixed point in (9). Furthermore, the scaling dimension of the scalar field to leading order in (18) agrees with that in (10). We conjecture that this equivalence holds in higher orders of perturbation theory and that the two models are in the same universality class.

Symmetry breaking in the model (11) can also be studied in the $1/N$ expansion by computing the effective potential for $\hat{\phi} \equiv \langle \phi \rangle$ which to leading order in N is readily obtained

from (13) as

$$\lim_{N \rightarrow \infty} \frac{1}{N} V_{\text{eff}}(\hat{\phi}) = \left(\frac{1}{\lambda} - \frac{1}{\lambda_C} \right) \frac{1}{2} \text{tr}_F \hat{\phi}^2 + 2 \frac{\Gamma(-D/2)}{(4\pi)^{D/2}} \text{tr}_F (\hat{\phi}^2)^{D/2} . \quad (19)$$

This potential clearly exhibits a second order phase transition at $\lambda \rightarrow \lambda_C$. When $\lambda > \lambda_C$, the eigenvalues of $\hat{\phi}$ obtain an expectation value with equal magnitude and undetermined sign. This symmetry breaking is consistent with the fact that $\text{tr} \hat{\phi} = 0$ only if N_F is even and the symmetry breaking pattern is $SU(N_F) \rightarrow SU(N_F/2) \otimes SU(N_F/2)$. (If N_F is odd, we can show for small values of N_F that the symmetry breaking pattern with $(N_F - 1)/2$ positive, $(N_F - 1)/2$ negative and one zero eigenvalue is preferred. In the following we shall avoid this complication by assuming that N_F is even.) If we assume this symmetry breaking pattern, it is straightforward to compute the leading correction to (19) when $D = 3$ and $\lambda = \lambda_C$. The 1-loop correction to the effective action is given by [22]

$$V_{\text{eff}}(\hat{\phi}) = \frac{1}{3\pi} N_F N |\hat{\phi}|^3 \left(1 - \frac{4}{\pi^2} \frac{N_F - 2}{N} \ln |\hat{\phi}|/\mu + \dots \right) \approx \frac{1}{3\pi} N N_F |\hat{\phi}|^{3 - \frac{2}{\pi^2} \frac{N_F - 2}{N}} . \quad (20)$$

The logarithmic corrections which are typical of next to leading order in the effective potential (and which can in some cases lead to a Coleman-Weinberg instability) exponentiate to give the scaling dimension of the potential [22]. Thus we have established that if N is large enough, the model has a second order phase transition. For large N_F the critical number of colors is $N^* = .27N_F$. At $N_F = 2$ $N^* = 34$, at $N_F = 4$ $N^* = 8$, at $N_F = 200$ $N^* = 55$. The behavior of the critical number of colors as a function of the number of flavors is consistent with results obtained in equivalent four fermi interaction theories [22]. It is interesting to speculate that the upper critical value of $N_F \sim 3.7N$ for existence of a chiral transition in the four-fermi theory is related to the upper critical $N_F \sim \text{const.} \cdot N_{\text{color}}$ for existence of chiral symmetry breaking in 2+1 dimensional QCD [30]. A correspondence of this kind was the intention of ref.[21]. However, if we naively identify $U(N)$ with color, that theory has either different symmetry or different degrees of freedom: If the QCD is confining, $U(N)$ is absent and if it is not confining, massless gluons should contribute to the critical behavior. Furthermore, in gauge theories at zero temperature, once N_F and N are given, either the vacuum is symmetric or the symmetry is spontaneously broken. There is no mechanical parameter that drives the symmetry breakdown unlike at finite temperature. Suppose an additional interaction such as the four-fermi coupling that we have considered in this paper is included in the theory. Then this type of effective low energy field theory might be applicable

for the gauged Nambu-Jona Lasinio model and our analysis would apply close to the Nambu-Jona-Lasinio fixed point where the gauge interaction is weak [28]. But in the region of phase space where the gauge interaction is strong a more refined analysis is needed. It has been seen that in this region the effective potential exhibits non-analytic behavior [29]. This issue warrants further study.

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Figure Captions

Figure 1: Fixed point values of the coupling constants g_1 and g_2 as N is varied from infinity where $(g_1^*, g_2^*) = (0, 0)$ to its lower critical value which is $N^* = 8$ for $N_F = 4$ (bottom curve), $N^* = 4$ for $N_F = 8$ (middle curve) and $N^* = 22$ for $N_F = 80$ (top curve).